Demand forecasting by temporal aggregation

The Impact on Supply Chain Cost

Bahman Rostami-Tabar [1] Stephen M. Disney [1]

[1] Logistics System Dynamics Group, Cardiff Business School, Cardiff University, Aberconway Building, Colum Drive, CF10 3EU, United Kingdom.

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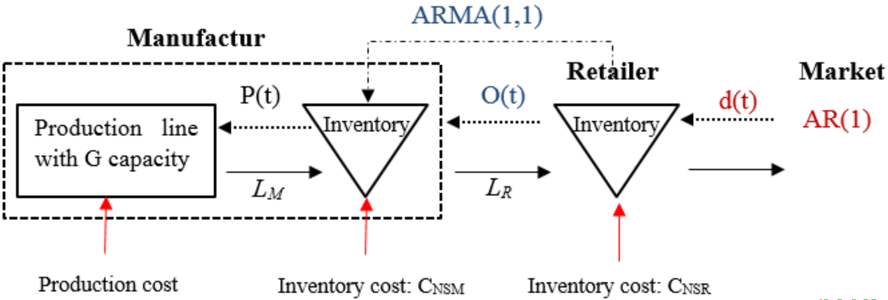
Supply Chain Model

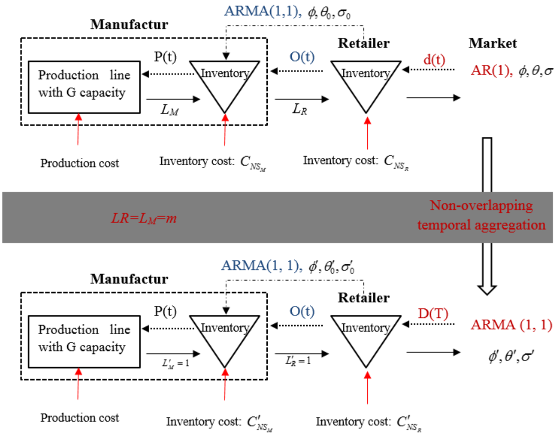
Minimizing the cost across the supply chain is one of the main concerns of SC managers.

One typical approach is to minimize the forecast error and consequently reduce unnecessary stock costs.



Problem

Should we use disaggregate or aggregated demand data in forecast in a SC setting?



Motivations for the current study

Recent advances have demonstrated the benefits of temporal aggregation for demand forecasting, including increased accuracy, improved stock control and reduced modeling uncertainty [Nikopoulos et al. (2011), Babai et al. (2012), Rostami-Tabar(2013, 2014), Kourentzes et al. (2017)].

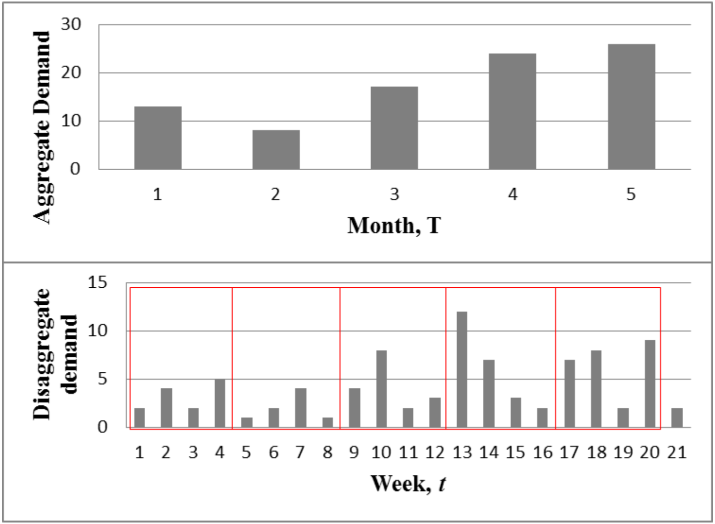
Substantial part of the literature in TA is dedicated to the impact of TA on forecast accuracy both empirically and analytically.

There is a lack of analytical investigation on conditions under which TA can reduce supply chain costs.

Our contribution

- We develop the variance and cost of inventory for a two echelon SC when the aggregated demand is used.
- We investigate the impact of the auto-regressive process parameter and the lead time/aggregation level on the variance and cost of inventory.
- We compare and contrast our results when aggregated demand is used in OUT policy to the case of using disaggregate demand.

Non-overlapping temporal aggregation



Assumptions

Data Generation Process: AR(1)

$$d(t) = C + \phi d(t-1) + \epsilon(t), \epsilon(t) \sim N(0, \sigma^2),$$

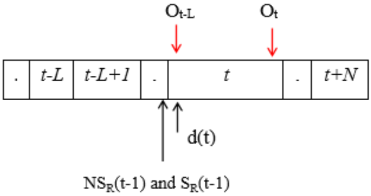
Forecasting method: Minimum Mean Squared Error (MMSE)

$$\hat{d}_{t+i} = E\left(d_{t+i} | d_t, d_{t\ 1}\right), i = 1, 2, ...$$

Inventory policy: OUT level policy.

Sequence of events

1. Inventory level at period $t-1$, $S_R(t-1)$ is observed.
2. Net stock at period $t-1$, $NS_R(t-1)$ is observed.
3. The order placed at period $(t-L)$ is received, $O(t-L)$.
4. Demand, $d(t)$ is satisfied from the stock.
5. The order $O(t)$ is calculated at the end of period t.
6. $NS_R(t)$ and consequently the amount of backorder $NS_R(t)^-$ and inventory $NS_R(t)^+$ are updated at the end of period t.
7. The cost is calculated for period t.



Ordering policy and cost at retailer

$$C_{NS_R} = h_R \times E\left(NS_R(t)^+\right) + b_R \times E\left(NS_R(t) \quad \right)$$

$$NS_R(t)^+ = max(0, NS_R(t))$$

$$NS_R(t) \quad = max(0, -NS_R(t))$$

$$NS_R(t) = NS_R(t-1) + O(t - L_R) - d(t)$$

$$O(t) = d(t) + S_R(t) - S_R(t-1)$$

$$S_R(t) = \hat{d}_{L_R}(t) + z_R \sigma_{NS_R}$$

Lead time forecast and inventory variance

$$\hat{d}_{L_R}(t) = E\left(\sum_{i=1}^{L_R} d(t+i)|d(t), d(t-1), ...\right) = \frac{1-\phi^{L_R}}{1-\phi}\left(\phi d(t) - \theta\epsilon_t\right)$$

$$\sigma^2{}_{NS_R} = E\left(Var\left(\sum_{i=1}^{L_R} d(t+i) \quad \hat{d}_{L_R}(t)\right)\right)$$

$$= \frac{L_R\left(\theta \quad 1\right)^2\left(\phi^2 \quad 1\right) + \left(\phi \quad \theta\right)\left(\phi^{L_R} \quad 1\right)\left(\theta\left(1+2\phi \quad \phi^{L_R}\right) + \phi\left(\phi^{L_R} \quad 1\right) \quad 2\right)}{\left(\phi \quad 1\right)^3\left(1+\phi\right)}\sigma_\epsilon^2$$

Relationship between aggregated and dis-aggregated parameters

The m periods non-overlapping aggregated demand($m = leadtime$), $D(T)$ can be expressed as a function of the non-aggregated demand series as follows

$$D(T+1) = \sum_{i=1}^{m} d(t+i),$$

$$D(T-k) = \sum_{i=1}^{m} d(t-i+k),$$

The aggregated process of AR(1) is an ARMA(1,1) with $D(T+1), D(T), ...$ follows an ARMA(1,1) with ϕ^0, θ^0 and σ^0

Relationship between aggregated and dis-aggregated parameters

$$\phi' = \phi^m$$

$$\theta' = \begin{cases} \dfrac{\left(X + \phi^{2m}X \quad 2\phi^m\right) + \sqrt{\left(X + \phi^{2m}X \quad 2\phi^m\right)^2 \quad 4\left(1 \quad \phi^m X\right)^2}}{2\left(1 \quad \phi^m X\right)}, & \phi > \theta \\ \dfrac{\left(X + \phi^{2m}X \quad 2\phi^m\right) \quad \sqrt{\left(X + \phi^{2m}X \quad 2\phi^m\right)^2 \quad 4\left(1 \quad \phi^m X\right)^2}}{2\left(1 \quad \phi^m X\right)}, & \phi < \theta \end{cases}$$

where

$$X = \frac{\left(m\gamma_0 + \gamma_1 \sum_{k=1}^{m-1} 2\left(m \quad k\right)\phi^k \quad 1\right.}{\gamma_1\left(\sum_{k=1}^{m} k\phi^k + \sum_{k=1}^{m-1} k\phi^{2m-k}\right)},$$

$$\sigma'^2 = \frac{\left(1 \quad \phi^{2m}\right)\left(m\gamma_0 + \gamma_1 \sum_{k=1}^{m-1} 2\left(m \quad k\right)\phi^{k-1}\right)}{\left(1 \quad 2\phi^m\theta' + \theta'^2\right)}$$

Leat time forecast and inventory variance with aggregated data

$$\hat{d}^{0}_{L_R}(t) = E\left(D(T+1)|D(T), D(T-1), ...\right) = \left(\phi^{0} D(T) - \theta^{0} \epsilon^{0}_{T}\right)$$

$$\begin{aligned} {\sigma^{2}}'_{NS_R} &= E\left(var\left(D(T+1) \quad \hat{d}'_{L_R}(t)\right)\right) \\ &= \frac{(\theta' \quad 1)^2 \left({\phi'}^2 \quad 1\right) + (\phi' \quad \theta')(\phi' \quad 1)(\theta'(1+2\phi' \quad \phi') + \phi'(\phi' \quad 1) \quad 2)}{(\phi' \quad 1)^3 (1+\phi')} {\sigma'_{\epsilon}}^2 \end{aligned}$$

Demand process at manufacturer

The OUT policy transforms an AR(1) demand process into the following ARMA(1,1) order process,

$$O(t+1) = C + \phi O(t) - \theta_0 \epsilon_0(t) + \epsilon_0(t+1), \epsilon_0(t) \sim N(0, \sigma_0^2),$$

$$\theta_0 = \frac{\phi\left(1-\phi^{L_R}\right)}{1-\phi^{L_R+1}}$$

$$\epsilon_0(t) = \left(1 + \frac{\phi\left(1-\phi^{L_R}\right)}{1-\phi}\right)\epsilon(t)$$

Ordering policy and cost at manufacturer

$$C_{NS_M} = h_M \times E\left(NS_M(t)^+\right) + b_M \times E\left(_M(t) \quad \right)$$

$$NS_M(t)^+ = max(0, NS_M(t))$$

$$NS_M(t) \quad = max(0, -NS_M(t))$$

$$NS_M(t) = NS_M(t-1) + P(t - L_M) - O(t)$$

$$P(t) = O(t) + S_M(t) - S_M(t-1)$$

$$S_M(t) = \hat{O}_{L_M}(t) + z_M \sigma_{NS_M}$$

Lead time forecast and inventory variance at manufacturer with disaggregate demand

$$\hat{d}_{L_M}(t) = E\left(\sum_{i=1}^{L_M} O(t+i)|O(t), O(t-1), ...\right) = \frac{1-\phi^{L_M}}{1-\phi}\left(\phi O(t) - \theta_0 \epsilon_{0,t}\right)$$

$$\sigma^2{}_{NS_M} = E\left(Var\left(\sum_{i=1}^{L_M} O(t+i) \quad \hat{O}_{L_M}(t)\right)\right)$$

$$= \frac{L_M(\theta \quad 1)^2(\phi \quad 1) + 2\phi^{L_M}(\theta \quad 1)(\phi \quad \theta)\left(\phi^{L_M} \quad 1\right) + \frac{(\theta-\phi)^2\phi^{2L_M}\left(\phi^{2L_M}-1\right)}{1+\phi}}{(\phi \quad 1)^3}\sigma_\epsilon^2$$

LSDG

Aggregated demand process at manufacturer

The OUT policy transforms an aggregated ARMA(1,1) demand process into another ARMA(1,1) order process, which is

$$O(T+1) = C + \phi^0 O(T) - \theta_0^0 \epsilon_0^0(t) + \epsilon_0^0(t+1), \epsilon_0(t) \sim N(0, \sigma_0^2),$$

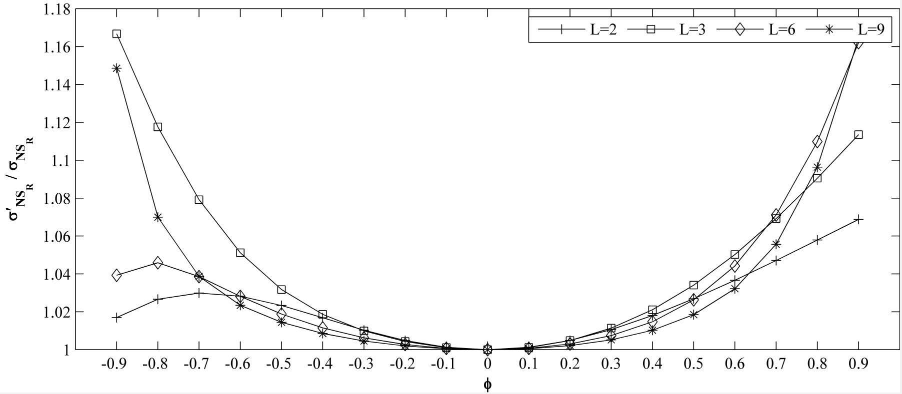
$$\theta_0^0 = \frac{\phi^0 \left(1 - {\phi^0}^{L_R}\right)}{1 - {\phi^0}^{L_R+1}}$$

$$\epsilon_0^0(t) = \left(1 + \frac{\phi^0 \left(1 - {\phi^0}^{L_R}\right)}{1 - \phi^0}\right) \epsilon^0(t)$$

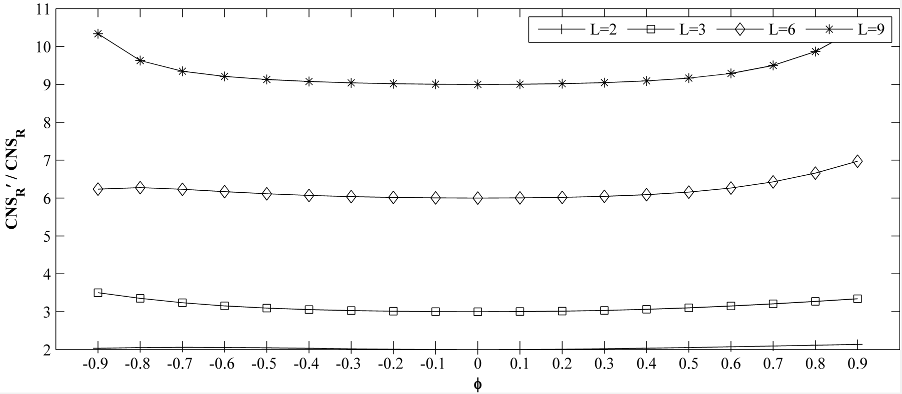
Lead time forecast and inventory variance at manufacturer with aggregated demand

$$\sigma'^2_{NS_M} = \frac{(\theta' \quad 1)^2 (\phi' \quad 1) + 2\phi' (\theta' \quad 1)(\phi' \quad \theta')(\phi' \quad 1) + \frac{(\theta' - \phi')^2 \phi'^2 \left(\phi'^2 - 1\right)}{1+\phi'}}{(\phi' \quad 1)^3} \sigma'^2_{\epsilon}$$

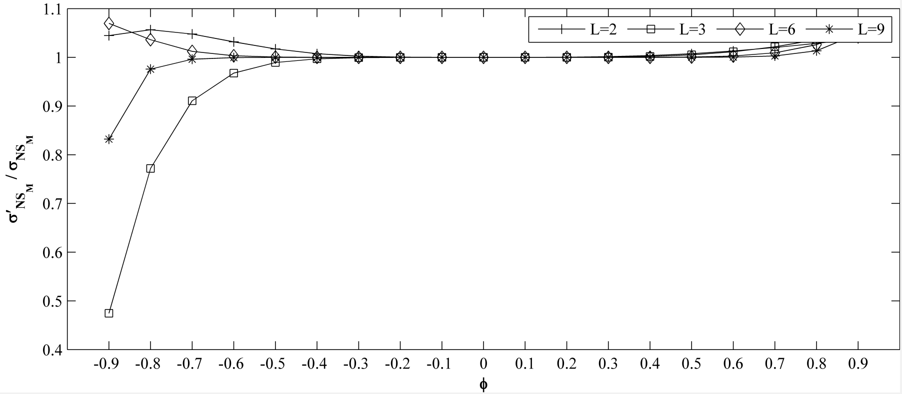
Ratio of inventory variance at retailer



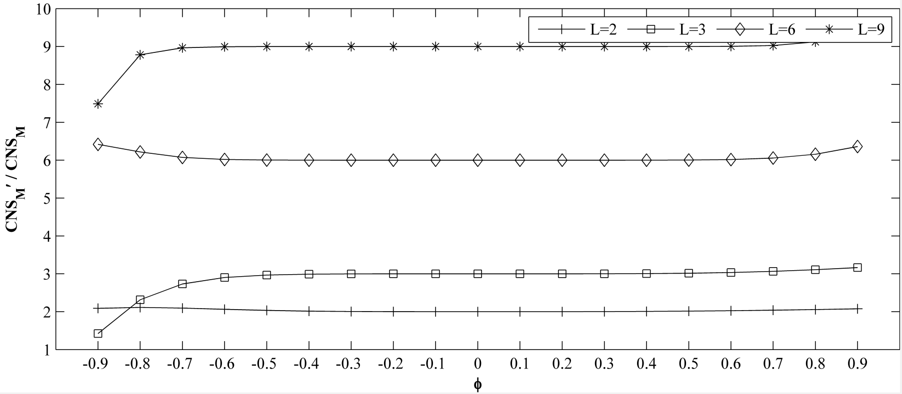
Ratio of inventory cost at retailer



Ratio of inventory variance at manufacturer



Ratio of inventory cost at manufacturer



Concluding remarks

TA does not improve inventory variance and cost when $\phi > 0$ regardless the value lead time/aggregation level values and supply chain stage

Inventory variance can be reduce at manufacturer level depending on the lead time/aggregation level values and only for $\phi < 0$

TA does not improve Inventory cost at manufacturer level

Benefits of using TA may be seen at manufacturer level with production cost.

Thank you for listening!