Demand forecasting by temporal aggregation

The Impact on Supply Chain Cost

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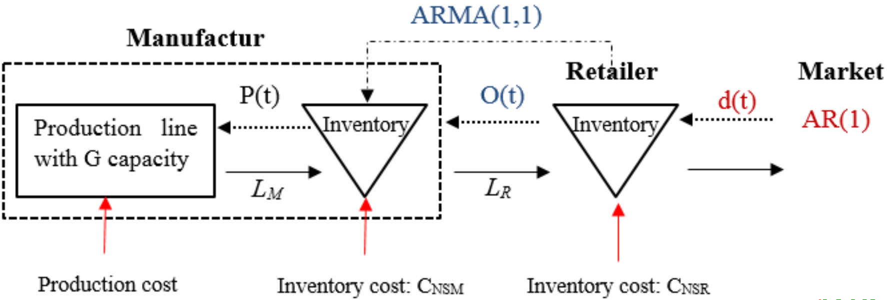
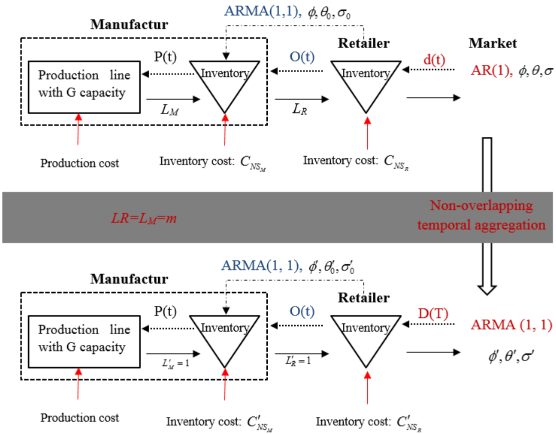
Supply Chain Model

- Minimizing the cost across the supply chain is one of the main concerns of SC managers.
- One typical approach is to minimize the forecast error and consequently reduce unnecessary stock costs.



Problem

Should we use disaggregate or aggregated demand data in forecast in a SC setting?



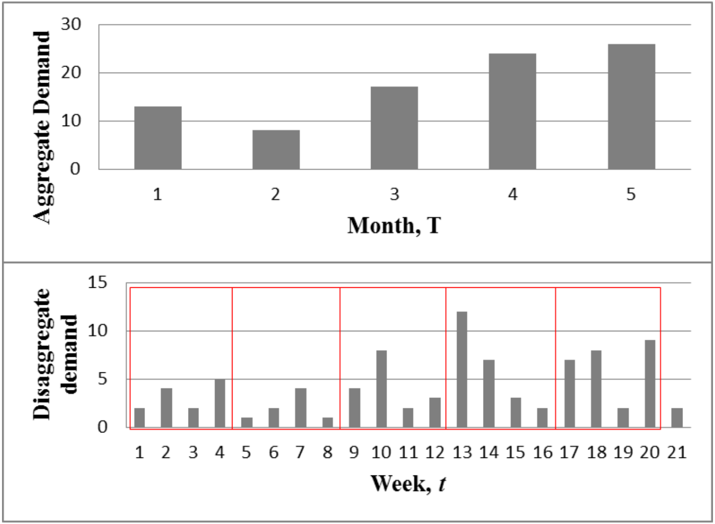
Motivations for the current study

- Recent advances have demonstrated the benefits of temporal aggregation for demand forecasting, including increased accuracy, improved stock control and reduced modeling uncertainty [Nikopoulos et al. (2011), Babai et al. (2012), Rostami-Tabar(2013, 2014), Kourentzes et al. (2017)].
- Substantial part of the literature in TA is dedicated to the impact of TA on forecast accuracy both empirically and analytically.
- There is a lack of analytical investigation on conditions under which TA can reduce supply chain costs.

Our contribution

- We develop the variance and cost of inventory for a two echelon SC when the aggregated demand is used.
- We investigate the impact of the auto-regressive process parameter and the lead time/aggregation level on the variance and cost of inventory.
- We compare and contrast our results when aggregated demand is used in OUT policy to the case of using disaggregate demand.

Non-overlapping temporal aggregation



Assumptions

- Data Generation Process: AR(1)

$$d(t) = C + \phi d(t-1) + \epsilon(t), \epsilon(t) \sim N(0, \sigma^2),$$

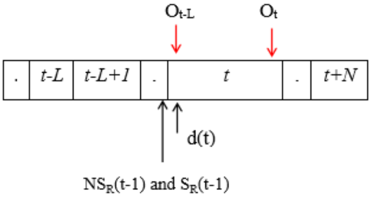
- Forecasting method: Minimum Mean Squared Error (MMSE)

$$\hat{d}_{t+i} = E\left(d_{t+i} | d_t, d_{t-1}\right), i = 1, 2, ...$$

- Inventory policy: OUT level policy.

Sequence of events

1. Inventory level at period $t-1$, $S_R(t-1)$ is observed.
2. Net stock at period $t-1$, $NS_R(t-1)$ is observed.
3. The order placed at period $(t-L)$ is received, $O(t-L)$.
4. Demand, $d(t)$ is satisfied from the stock.
5. The order $O(t)$ is calculated at the end of period t.
6. $NS_R(t)$ and consequently the amount of backorder $NS_R(t)^-$ and inventory $NS_R(t)^+$ are updated at the end of period t.
7. The cost is calculated for period t.



Ordering policy and cost at retailer

$$C_{NS_R} = h_R \times E\left(NS_R(t)^+\right) + b_R \times E\left(NS_R(t)^-\right)$$

$$NS_R(t)^+ = max(0, NS_R(t))$$

$$NS_R(t)^- = max(0, -NS_R(t))$$

$$NS_R(t) = NS_R(t-1) + O(t-L_R) - d(t)$$

$$O(t) = d(t) + S_R(t) - S_R(t-1)$$

$$S_R(t) = \hat{d}_{L_R}(t) + z_R \sigma_{NS_R}$$

Lead time forecast and inventory variance

$$\hat{d}_{L_R}(t) = E\left(\sum_{i=1}^{L_R} d(t+i) | d(t), d(t-1), ...\right) = \frac{1-\phi^{L_R}}{1-\phi}\left(\phi d(t) - \theta\epsilon_t\right)$$

$$\sigma^2{}_{NS_R} = E\left(Var\left(\sum_{i=1}^{L_R} d(t+i) - \hat{d}_{L_R}(t)\right)\right)$$
$$= \frac{L_R(\theta-1)^2\left(\phi^2-1\right) + (\phi-\theta)\left(\phi^{L_R}-1\right)\left(\theta\left(1+2\phi-\phi^{L_R}\right)+\phi\left(\phi^{L_R}-1\right)-2\right)}{(\phi-1)^3(1+\phi)}\sigma_\epsilon^2$$

Relationship between aggregated and dis-aggregated parameters

The m periods non-overlapping aggregated demand($m = leadtime$), $D(T)$ can be expressed as a function of the non-aggregated demand series as follows

$$D(T+1) = \sum_{i=1}^{m} d(t+i),$$

$$D(T-k) = \sum_{i=1}^{m} d(t-i+k),$$

- The aggregated process of AR(1) is an ARMA(1,1) with
- $D(T+1), D(T), \ldots$ follows an ARMA(1,1) with ϕ', θ' and σ'

Relationship between aggregated and dis-aggregated parameters

$$\phi' = \phi^m$$

$$\theta' = \begin{cases} \dfrac{-\left(X + \phi^{2m} X - 2\phi^m\right) + \sqrt{\left(X + \phi^{2m} X - 2\phi^m\right)^2 - 4\left(1 - \phi^m X\right)^2}}{2\left(1 - \phi^m X\right)}, & \phi > \theta \\ \dfrac{-\left(X + \phi^{2m} X - 2\phi^m\right) - \sqrt{\left(X + \phi^{2m} X - 2\phi^m\right)^2 - 4\left(1 - \phi^m X\right)^2}}{2\left(1 - \phi^m X\right)}, & \phi < \theta \end{cases}$$

where

$$X = \frac{\left(m\gamma_0 + \gamma_1 \sum_{k=1}^{m-1} 2\left(m-k\right)\phi^k - 1\right.}{\gamma_1 \left(\sum_{k=1}^{m} k\phi^k + \sum_{k=1}^{m-1} k\phi^{2m-k}\right)},$$

$$\sigma'^2 = \frac{\left(1 - \phi^{2m}\right)\left(m\gamma_0 + \gamma_1 \sum_{k=1}^{m-1} 2\left(m-k\right)\phi^{k-1}\right)}{\left(1 - 2\phi^m\theta' + \theta'^2\right)}$$

Leat time forecast and inventory variance with aggregated data

$$\hat{d}'_{L_R}(t) = E\left(D(T+1) | D(T), D(T-1), ...\right) = \left(\phi' D(T) - \theta' \epsilon'_T\right)$$

$$\begin{aligned} {\sigma^{2}}'_{NS_R} &= E\left(var\left(D(T+1) - \hat{d}'_{L_R}(t)\right)\right) \\ &= \frac{(\theta'-1)^2\left({\phi'}^2-1\right)+(\phi'-\theta')(\phi'-1)\left(\theta'(1+2\phi'-\phi')+\phi'(\phi'-1)-2\right)}{(\phi'-1)^3(1+\phi')}{\sigma'_\epsilon}^2 \end{aligned}$$

Demand process at manufacturer

- The OUT policy transforms an AR(1) demand process into the following ARMA(1,1) order process,

$$O(t+1) = C + \phi O(t) - \theta_0 \epsilon_0(t) + \epsilon_0(t+1), \epsilon_0(t) \sim N(0, \sigma_0^2),$$

$$\theta_0 = \frac{\phi\left(1 - \phi^{L_R}\right)}{1 - \phi^{L_R+1}}$$

$$\epsilon_0(t) = \left(1 + \frac{\phi\left(1 - \phi^{L_R}\right)}{1 - \phi}\right)\epsilon(t)$$

LSDG

Ordering policy and cost at manufacturer

$$C_{NS_M} = h_M \times E\left(NS_M(t)^+\right) + b_M \times E\left(_M(t)^-\right)$$

$$NS_M(t)^+ = max(0, NS_M(t))$$

$$NS_M(t)^- = max(0, -NS_M(t))$$

$$NS_M(t) = NS_M(t-1) + P(t - L_M) - O(t)$$

$$P(t) = O(t) + S_M(t) - S_M(t-1)$$

$$S_M(t) = \hat{O}_{L_M}(t) + z_M \sigma_{NS_M}$$

Lead time forecast and inventory variance at manufacturer with disaggregate demand

$$\hat{d}_{L_M}(t) = E\left(\sum_{i=1}^{L_M} O(t+i)|O(t), O(t-1), ...\right) = \frac{1-\phi^{L_M}}{1-\phi}\left(\phi O(t) - \theta_0 \epsilon_{0,t}\right)$$

$$\sigma^2{}_{NS_M} = E\left(Var\left(\sum_{i=1}^{L_M} O(t+i) - \hat{O}_{L_M}(t)\right)\right)$$
$$= \frac{L_M\left(\theta-1\right)^2\left(\phi-1\right) + 2\phi^{L_M}\left(\theta-1\right)\left(\phi-\theta\right)\left(\phi^{L_M}-1\right) + \frac{(\theta-\phi)^2\phi^{2L_M}\left(\phi^{2L_M}-1\right)}{1+\phi}}{\left(\phi-1\right)^3}\sigma_\epsilon^2$$

Aggregated demand process at manufacturer

- The OUT policy transforms an aggregated ARMA(1,1) demand process into another ARMA(1,1) order process, which is

$$O(T+1) = C + \phi' O(T) - \theta'_0 \epsilon'_0(t) + \epsilon'_0(t+1), \epsilon_0(t) \sim N(0, \sigma_0^2),$$

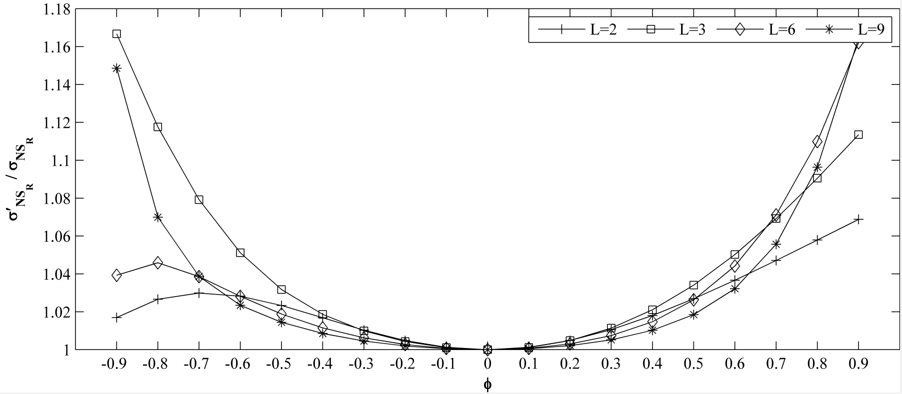
$$\theta'_0 = \frac{\phi'\left(1 - {\phi'}^{L_R}\right)}{1 - {\phi'}^{L_R+1}}$$

$$\epsilon'_0(t) = \left(1 + \frac{\phi'\left(1 - {\phi'}^{L_R}\right)}{1 - \phi'}\right) \epsilon'(t)$$

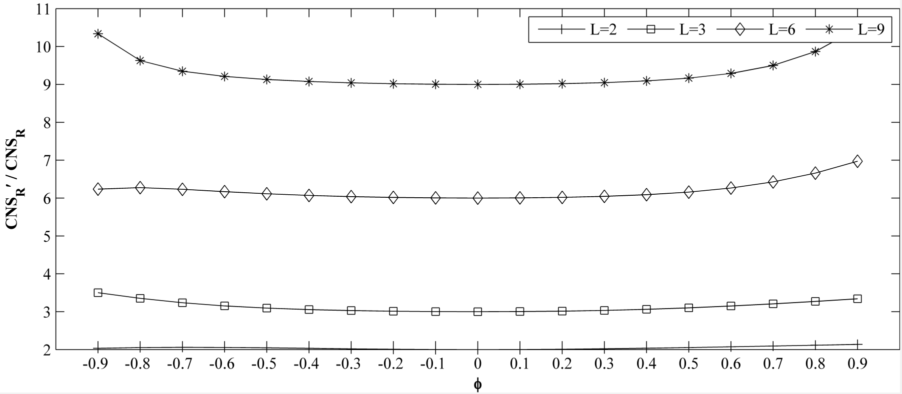
Lead time forecast and inventory variance at manufacturer with aggregated demand

$$\sigma'^2_{NS_M} = \frac{(\theta' - 1)^2 (\phi' - 1) + 2\phi' (\theta' - 1) (\phi' - \theta') (\phi' - 1) + \frac{(\theta' - \phi')^2 \phi'^2 \left(\phi'^2 - 1\right)}{1 + \phi'}}{(\phi' - 1)^3} \sigma'^2_{\epsilon}$$

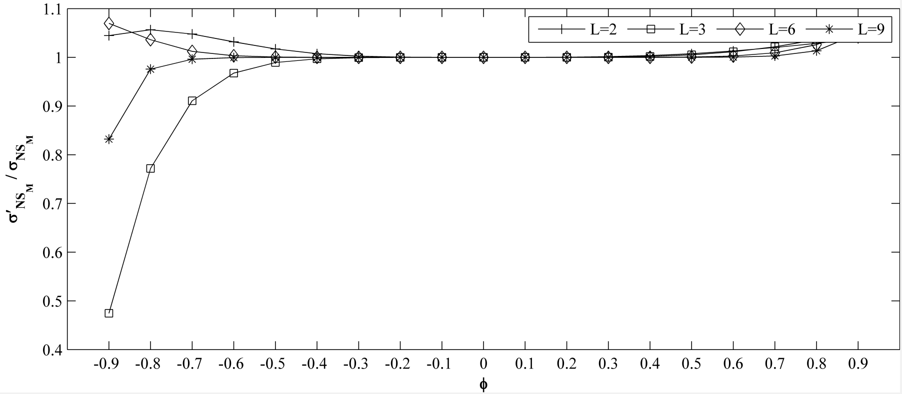
Ratio of inventory variance at retailer



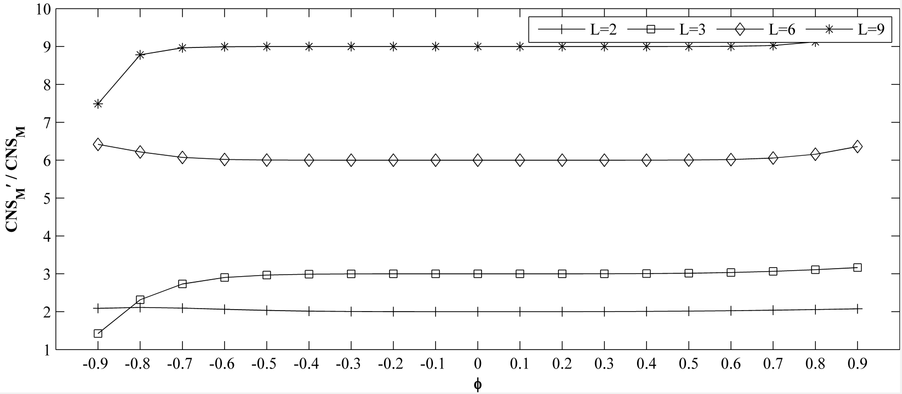
Ratio of inventory cost at retailer



Ratio of inventory variance at manufacturer



Ratio of inventory cost at manufacturer



Concluding remarks

- TA does not improve inventory variance and cost when $\phi > 0$ regardless the value lead time/aggregation level values and supply chain stage
- Inventory variance can be reduce at manufacturer level depending on the lead time/aggregation level values and only for $\phi < 0$
- TA does not improve Inventory cost at manufacturer level
- Benefits of using TA may be seen at manufacturer level with production cost.

Thank you for listening!