The more, the better, usually: An analytical investigation of the yield rate paradox in closed-loop supply chains

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Motivation:

Sustainability Perspective: Aquafil Group

"the first rule in being sustainable is being profitable, because if you are not profitable as a company, sooner or later you go out of business, and yes, you aren't damaging the environment - but also you don't exist anymore."

Mr. Giulio Bonazzi, CEO and President of Aquafil Group

https://www.econyl.com/assets/uploads/EllenMacArthur_case-study-aquafil-group.pdf

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Literature Review

Higher return (yield) positively impacts on the dynamics of CLSC

- [Tang and Naim, 2004]
- [Zhou and Disney, 2006]
- [Cannella et al., 2016]
- [Zhou et al., 2017]
- [Ponte et al., 2019]

Impacts of return yield may vary depending on parameters

- [Hosoda et al., 2015]

A comprehensive review of the CLSC literature:
[Akçalı and Çetinkaya, 2011], [Govindan et al., 2015], [Cannella et al., 2016] and [Goltsos et al., 2018].

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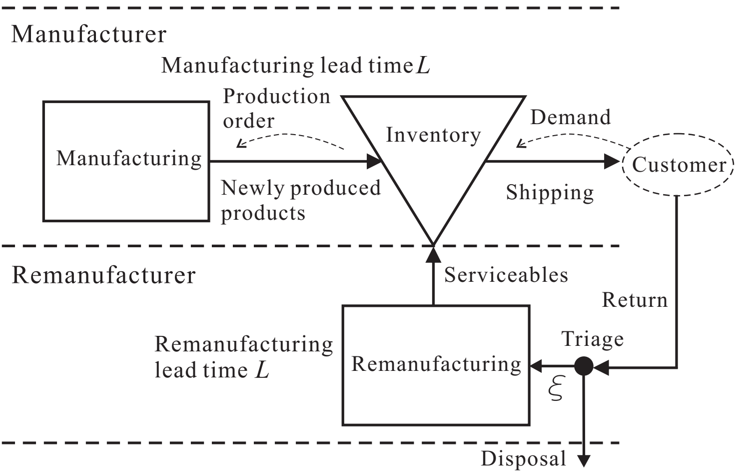
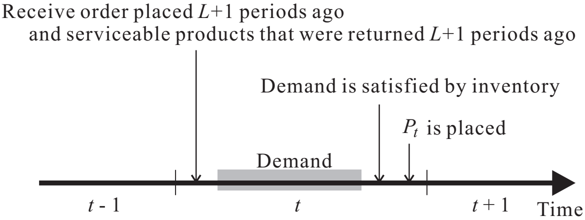
Model

Schematic of Model: Remanufactured products are considered as good as new.



Model

Sequence of Events and the Inventory Balance Equation at the Manufacturer



$$NS_t = NS_{t-1} + \overbrace{\xi \; R_{t-(L+1)}}^{\textit{Serviceable products}} + P_{t-(L+1)} - D_t$$

$$\underbrace{NS_t}_{\textit{Net stock}} \quad \underbrace{\xi}_{\textit{Yield rate}} \quad \underbrace{R_{t-(L+1)}}_{\textit{Return}} \quad \underbrace{P_{t-(L+1)}}_{\textit{Production order}} \quad \underbrace{D_t}_{\textit{Demand}}$$

where

- L: Manufacturing and remanufacturing lead times

Vector Autoregressive Demand and Return Model

$$\begin{bmatrix} D_t \\ R_t \end{bmatrix} = \begin{bmatrix} \mu_d \\ \mu_r \end{bmatrix} + \begin{bmatrix} \phi_d & 0 \\ \theta_r & \phi_r \end{bmatrix} \begin{bmatrix} D_{t-1} - \mu_d \\ R_{t-1} - \mu_r \end{bmatrix} + \begin{bmatrix} \varepsilon_{d,t} \\ \varepsilon_{r,t} \end{bmatrix}$$

- VAR model origin:
  - [Box and Tiao, 1977] [Tiao and Box, 1981]
- Application of VAR model to CL supply chain
  - [Hosoda and Disney, 2018]
- D_t: Demand at time period t
- R_t: Returns at time period t
- μ_d, μ_r: Mean of the demand and the returns, respectively
- ϕ_d, ϕ_r: Auto-correlation coefficients for the demand and the returns, respectively
- θ_r: Cross-correlation coefficient between returns and previous demand
- $\varepsilon_{d,t}$, $\varepsilon_{r,t}$: i.i.d. random variables with zero mean and constant SDs σ_d and σ_r, respectively
- Demand is not correlated with previous returns

Vector Autoregressive Demand and Return Model

Variances and Covariances

$$\mathbb{V}[D] = \frac{\sigma^2_{\varepsilon_d}}{1-\phi_d^2}$$

$$\mathbb{V}[R] = \frac{\theta_r^2(1+\phi_d\phi_r)\mathbb{V}[D]}{(1-\phi_d\phi_r)(1-\phi_r^2)} + \frac{\sigma^2_{\varepsilon_r}}{1-\phi_r^2}$$

$$COV_0 = \mathbb{E}[(D_t-\mu_d)(R_t-\mu_r)] = \frac{\phi_d\theta_r\mathbb{V}[D]}{1-\phi_d\phi_r}$$

$$COV_{L+1} = \mathbb{E}[(D_t-\mu_d)(R_{t-(L+1)}-\mu_r)] = \phi_d^{L+1}COV_0.$$

Closed-Loop Order-Up-To (CL-OUT) Policy

$$P_t = \mathbb{E}\left[D_{t+L+1}\right] - \xi R_t + TNS - NS_t + \mathbb{E}\left[\sum_{i=1}^{L} D_{t+i}\right] - \left(WIP_t + WIPR_t\right),$$

where

- $\mathbb{E}\left[D_{t+L+1}\right] - \xi R_t$: Forecast of products required at time $t+L+1$.
- $TNS - NS_t$: Inventory feedback loop.
- $\mathbb{E}\left[\sum_{i=1}^{L} D_{t+i}\right] - \left(WIP_t + WIPR_t\right)$: Total WIP feedback loop.

CL-OUT Policy

Variances of the net stock levels $\mathbb{V}[NS]$ and production of new items $\mathbb{V}[P]$

$$\mathbb{V}[NS] = \frac{(L+1)(1-\phi_d^2) + \phi_d(1-\phi_d^{L+1})(\phi_d^{L+2} - \phi_d - 2)}{(1-\phi_d)^2(1-\phi_d^2)}\sigma_{\varepsilon_d}^2$$

$$\mathbb{V}[P] = \mathbb{V}[D] - 2\xi\frac{\theta_r\phi_d^{L+2}}{1-\phi_d\phi_r}\mathbb{V}[D] + \xi^2\mathbb{V}[R] + 2\frac{\phi_d(1-\phi_d^{L+1})(1-\phi_d^{L+2})}{(1-\phi_d^2)(1-\phi_d)}\sigma_{\varepsilon_d}^2$$

- The variance of the net stock levels, $\mathbb{V}[NS]$, is independent of the re-manufacturing parameters, $\{\xi,\ \phi_r,\ \theta_r,$ and $\sigma_{\varepsilon_r}\}$.
- Thus, inventory related costs are not influenced by the decision variable ξ and the uncertainty in the returns, $\varepsilon_{r,t}$.

Objective Cost Function: System-wide Cost

$$C(\xi) = \mathbb{E}[h(NS_t)^+ + b(-NS_t)^+] +$$
$$\min_{\xi} \Big\{ \underbrace{\mathbb{E}[uk + w(P_t - k)^+] + \mathbb{E}[u'k' + w'(\xi R_t - k')^+] + \mathbb{E}[(1-\xi)R_t \times g]}_{\text{Function of } \xi} \Big\},$$

where

- ξ: yield rate (decision variable)
- h: unit holding cost, b: unit backlog cost.
- u: unit manufacturing cost when P_t is less than the capacity k.
- w: unit manufacturing cost when P_t is greater than the capacity k.
- u': unit remanu. cost when ξR_t is less than the capacity k'.
- w': unit remanu. cost when ξR_t is greater than the capacity k'.
- $u > u'$ and $w > w'$.
- g: unit disposal cost.

Derivatives of Objective Cost Function

First and Second derivatives

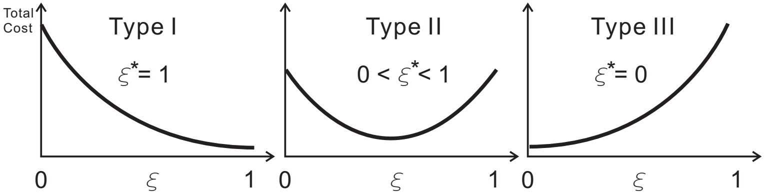
$$\frac{\partial C(\xi)}{\partial \xi} = w\varphi\left[\Phi^{-1}\left[\frac{w-u}{w}\right]\right]\frac{\xi\mathbb{V}[R]-COV_{L+1}}{\sqrt{\mathbb{V}[P]}} + w'\varphi\left[\Phi^{-1}\left[\frac{w'-u'}{w'}\right]\right]\sqrt{\mathbb{V}[R]}-(u-u')\mu_r - g\mu_r.$$

$$\frac{\partial^2 C(\xi)}{\partial \xi^2} = w\varphi\left[\Phi^{-1}\left[\frac{w-u}{w}\right]\right]\frac{\mathbb{V}[R]\mathbb{V}[P|\xi=0]-(COV_{L+1})^2}{\mathbb{V}[P]\sqrt{\mathbb{V}[P]}} > 0, \text{if } 1 > \phi_d \geq 0.$$

Convexity of Objective Cost Function

Corollary

When the CL-OUT policy is exploited by the manufacturer, the system-wide cost is convex in the triage yield, ξ.



"Yield Rate Paradox" could exist

System-wide cost could increase in the triage yield rate ξ, even though the unit remanufacturing cost is lower than that of the manufacturing.

First-order derivative is decreasing in μ_r as long as $u > u'$

Corollary

When the CL-OUT policy is exploited by the manufacturer, the first-order derivative of the system-wide cost is decreasing in μ_r as long as $u > u'$.

$$\frac{\partial C(\xi)}{\partial \xi} = w\varphi\left[\Phi^{-1}\left[\frac{w-u}{w}\right]\right]\frac{\xi\mathbb{V}[R] - COV_{L+1}}{\sqrt{\mathbb{V}[P]}} + w'\varphi\left[\Phi^{-1}\left[\frac{w'-u'}{w'}\right]\right]\sqrt{\mathbb{V}[R]} - (u-u')\mu_r - g\mu_r.$$

Larger average returns could help profitability

Larger average returns might shift the cost function curve from Type III to Type II (or, from Type II to Type I).

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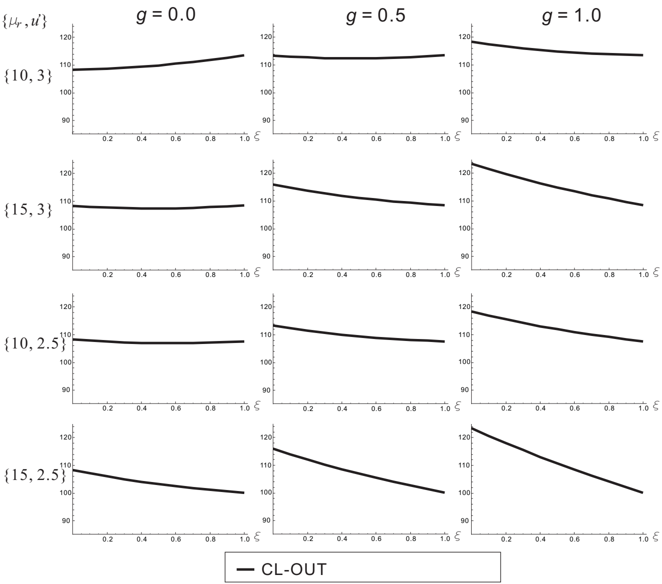
Example of Numerical Analysis: Data Set

- Demand
  - $\mu_d = 20$
  - $\phi_d = 0.4$
  - $\sigma_{\varepsilon_d} = 3$
- Return
  - $\mu_r = \{10, 15\}$
  - $\phi_r = 0.7$
  - $\theta_r = 0.5$
  - $\sigma_{\varepsilon_r} = 1$
- Lead time
  - $L = 1$

- Inventory related cost
  - $h = 1$
  - $b = 9$
- Cost of manufacturing new products
  - $w = 11$
  - $u = 4$
- Remanufacturing cost
  - $w' = 9$
  - $u' = \{3, 2.5\}$
- Disposal cost
  - $g = \{0, 0.5, 1\}$

Impact of μ_r, u', g with CL-OUT Policy



Closed-Loop APIOBPCS (CL-APIOBPCS)

Automatic Pipeline Inventory Order Based Production Control System for Closed-Loop Supply Chain

A modified OUT policy with feedback proportional controllers [John et al., 1994].

$$
\begin{aligned}
P_t = {} & \mathbb{E}\left[D_{t+L+1}\right] - \xi R_t \\
& + (1-\alpha)(TNS - NS_t) \\
& + (1-\alpha)\left(\mathbb{E}\left[\sum_{i=1}^{L} D_{t+i}\right] - \left(WIP_t + WIPR_t\right)\right),
\end{aligned}
$$

where $|\alpha| < 1$. Note that when $\alpha = 0$, CL-APIOBPCS degenerates into the CL-OUT policy.
[Lin et al., 2017] provides a comprehensive review of the APIOBPCS family of production planning archetypes.

CL-APIOBPCS

Variances of the net stock levels $\mathbb{V}_A[NS]$ and production of new items $\mathbb{V}_A[P]$

$$\mathbb{V}_A[NS] = \mathbb{V}[NS] + \frac{(\phi_d^{L+1} - 1)^2 \alpha^2 \sigma_{\varepsilon_d}^2}{(\phi_d - 1)^2 (1 - \alpha^2)} \geq \mathbb{V}[NS].$$

$$\mathbb{V}_A[P] = \mathbb{V}[P] + 2\alpha\sigma_{\varepsilon_d}^2 \left(\phi_d^{L+1} - 1\right) \times \frac{\begin{pmatrix} \xi\theta_r(\alpha^2 - 1)(\phi_d - 1) \\ +\left(\phi_d(\alpha - \phi_d^L(\alpha - \phi_d + \phi_d(\phi_d - 1)(1 + \alpha))) - 1\right)(\alpha\phi_r - 1) \end{pmatrix}}{(1 + \alpha)(\phi_d - 1)^2(\alpha\phi_d - 1)(\alpha\phi_r - 1)}.$$

- $\mathbb{V}_A[NS]$ is still independent of the re-manufacturing parameters, $\{\xi,\ L,\ \phi_r,\ \theta_r$, and $\sigma_{\varepsilon_r}\}$ as in the case of CL-OUT policy.
- It is easy to recognize that $\mathbb{V}_A[NS] = \mathbb{V}[NS]$ and $\mathbb{V}_A[P] = \mathbb{V}[P]$ given that $\alpha = 0$.

Advantage of CL-APIOBPCS

- Objective cost function for CL-OUT policy:

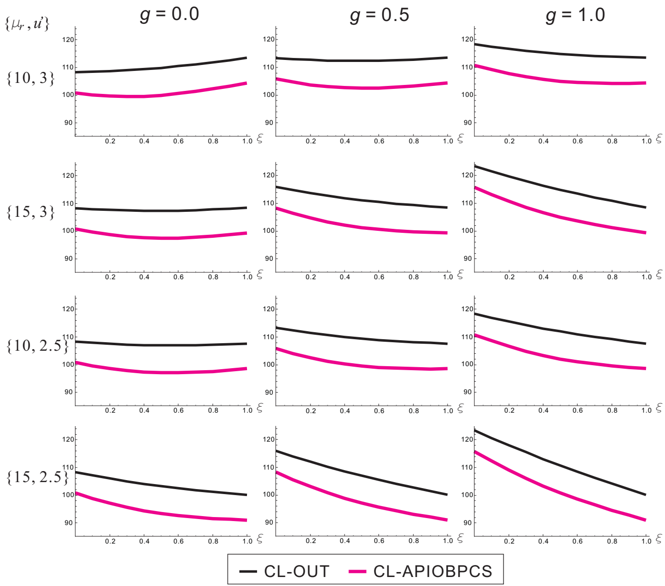
$$C(\xi) = \mathbb{E}[h(NS_t)^+ + b(-NS_t)^+] + \min_{\xi} \Big\{ \underbrace{\mathbb{E}[uk + w(P_t - k)^+] + \mathbb{E}[u'k' + w'(\xi R_t - k')^+] + \mathbb{E}[(1-\xi)R_t \times g]}_{\text{Function of } \xi} \Big\}$$

- Objective cost function for CL-APIOBPCS policy:

$$\begin{aligned} C(\alpha, \xi) = \min_{\alpha,\, \xi} \Big\{ & \mathbb{E}[h(NS_t)^+ + b(-NS_t)^+] \\ & + \mathbb{E}[uk + w(P_t - k)^+] \\ & + \mathbb{E}[u'k' + w'(\xi R_t - k')^+] \\ & + g\mathbb{E}[(1-\xi)R_t] \Big\} \end{aligned}$$

The first two terms are marked "Function of α"; the last three terms are marked "Function of ξ".

CL-APIOBPCS can reduce the cost further



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Summary and Conclusion

Major Contributions

Existence of Yield Rate Paradox and its Mitigation Policies

- System-wide cost is convex in the triage yield rate ξ.
- Larger μ_r and/or smaller u' can mitigate *yield rate paradox*.
- Higher disposal cost g can induce one to remaufacture all returns.
- Introduce CL-APIOBPCS
  - Proportional controller α can reduce the cost further.

Social/Managerial Implication

- Larger mean of the returns is good for not only the sustainability but also a cost-conscious system.

Future Research Direction

- When/how does the yield rate paradox actually emerge in real life?
- Are there any other mitigation policies?

Thank you for your attention. Any questions?

References I

Akçalı, E. and Çetinkaya, S. (2011).
Quantitative models for inventory and production planning in closed-loop supply chains.
International Journal of Production Research, 49:2373–2407.

Box, G. E. P. and Tiao, G. C. (1977).
A canonical analysis of multiple time series.
Biometrika, 64:355–365.

Cannella, S., Bruccoleri, M., and Framinan, J. M. (2016).
Closed-loop supply chains: What reverse logistics factors influence performance?
International Journal of Production Economics, 175:35–49.

Goltsos, T. E., Ponte, B., Wang, S., Liu, Y., Naim, M. M., and Syntetos, A. A. (2018).
The boomerang returns? accounting for the impact of uncertainties on the dynamics of remanufacturing systems.
International Journal of Production Research, In press.

Govindan, K., Soleimani, H., and Kannan, D. (2015).
Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future.
European Journal of Operational Research, 240:603–626.

Hosoda, T. and Disney, S. M. (2018).
A unified theory of the dynamics of closed-loop supply chains.
European Journal of Operational Research, 269(1):313–326.

Hosoda, T., Disney, S. M., and Gavirneni, S. (2015).
The impact of information sharing, random yield, correlation, and lead times in closed loop supply chains.
European Journal of Operational Research, 246:827–836.

References II

John, S., Naim, M. M., and Towill, D. R. (1994).
Dynamic analysis of a wip compensated decision support system.
International Journal of Manufacturing System Design, 1(4):283–297.

Lin, J., Naim, M. M., Purvis, L., and Gosling, J. (2017).
The extension and exploitation of the inventory and order based production control system archetype from 1982 to 2015.
International Journal of Production Economics, 194:135–152.

Ponte, B., Naim, M. M., and Syntetos, A. A. (2019).
The effect of returns volume uncertainty on the dynamic performance of closed-loop supply chains.
Journal of Remanufacturing, In press.

Tang, O. and Naim, M. M. (2004).
The impact of informatin transparency on the dynamic behaviour of a hybrid manufacturing/remanufacturing system.
International Journal of Production Research, 42(19):4135–4152.

Tiao, G. C. and Box, G. E. P. (1981).
Modeling multiple times series with applications.
Journal of the American Statistical Association, 76:802–816.

Zhou, L. and Disney, S. M. (2006).
Bullwhip and inventory variance in a closed loop supply chain.
OR Spectrum, 28:127–149.

Zhou, L., Naim, M. M., and Disney, S. M. (2017).
The impact of product returns and remanufacturing uncertainties on the dynamic performance of a multi-echelon closed-loop supply chain.
International Journal of Production Economics, 183:487–502.